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ON

THE BASIS OF MUSIC,

CONTAINING AN ELEMENTARY ACCOUNT OF

THE NATURE OF MUSICAL NOTES AND CHORDS,
THE GENERATION OF SCALES AND MODULATIONS,
AND
THE ORIGIN AND EFFECTS OF THE USUAL
TEMPERED SCALES.

BY

ALEXANDER J. ELLIS, B.A., F.R.S.,

TRANSLATOR OF HELMHOLTZ'S GREAT WORK "ON THE SENSATIONS OF TONE."

A LECTURE
DELIVERED BEFORE THE COLLEGE OF PRECEPTORS,
14 MARCH, 1877.

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*Reprinted, with numerous Additions and Alterations, from
the Educational Times for April,* 1877.

LONDON:
C. F. HODGSON & SON, GOUGH SQUARE,
FLEET STREET.

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1872, Feb. 1.
Gift of
Francis J. Child,
Prof. in Harv. Coll.
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THE BASIS OF MUSIC.

1. *Troublesome Questions from Learners.*—What is Music? A college friend of mine, a great mathematician, declared that it was "the most endurable of noises." But is music a noise? If not, how do you distinguish noise from music? and may there not be, is not there often, very noisy music? And what are dissonances but noises? Yet are not dissonances a recognised part of music?

Another friend of mine, a lady, wrote a book shewing how "to teach a child music." When I came to examine it, I found it was a plan she had thought out for teaching a little girl to play on the piano. But is music identical with the piano, even supposing it to be properly played? Surely there was music in the world before it dreamed of pianos, which are quite modern inventions. Indeed, they did not come into use till the last third of the 18th century.* I am afraid we are too apt to identify music with the sounds to be elicited from a piano. We associate the musical scale with successions of dozens of black and white keys, five of the former and seven of the latter. Many young ladies "try a song" by playing the notes first on their pianos, and imitating their sounds. But do they imitate them? Are the sounds they utter in any respect like those of the piano, except in being—more or less—of the same pitch. What's the cause of the difference of effect? And what's the meaning of sameness of pitch? And, to go back, is the pianoforte scale of pitches a heaven-imparted gift which always was, and always will be? And if not, how did it come about?

What's the meaning of the piano being "out of tune"? How in the world does the tuner know when he has got it in tune? Does he really know, or does he only guess? Can the young lady tune her own harp, or guitar, if she ventures beyond the piano? and, if so, when and how and why is "her ear satisfied"? And then there's the fiddler and his tribe, whom you hear at concerts, and who makes a frightful noise when he "tunes up"; does *he* judge in the same way? And how does he know how to "stop" rightly, when he has no "frets" to guide him, as the guitarrist, or single strings and keys, like the harpist and pianist? Why *should* he put his fingers in one place rather than another? Why is his scraping frightful at times? Why is an harmonium so gritty in tone? Why does it sound, note for note, of the same pitch as the piano to which it is tuned, and yet make all the chords so ugly? And, by-the-bye, what is a "chord"? Why are some notes suitable for a chord and others not? Are they not all musical notes?

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* See Bosanquet on Temperament (1876), p. 31.

What induces composers to put in those horrid "accidentals," which cause the young lady so much more trouble to play than they are worth, so far as she can see? And why are pieces written in different keys? Why do we have "signatures of five flats and six sharps," which puzzle one so? And why call a note B at one time and C flat at another? or C at one time and B sharp at another? What *is* the use of having two or three names for every note, it *is* so puzzling? And, please, why are there no black notes between E and F, or between B and C on the piano? And why can you play Scotch tunes by playing on the black notes only?

2. *The Teacher should be able to answer them.*—Perhaps those are questions enough to shew what I mean by the Basis of Music. All of them must have occurred to the great mass of girls who learn the piano, if they have anything of an inquiring disposition. And I think that every teacher of music should be able to give some sort of answer. As, however, there are very few works either accessible or intelligible to the mass of musical men, which convey the necessary information, and as it so happens that I have had to attend to the subject a good deal during the last few years,* and am therefore able to shew you, or rather make you hear, some things which will tend to give you a notion of the facts that underlie the answers to the shoal of questions which I have suggested, I will endeavour to give an epitome of the facts recently brought to light, especially by Prof. Helmholtz. I am not going to answer all the questions I put, or to give you any full and precise explanation; that would lie far beyond the limits of a lecture. But I wish to shew you a few simple experiments, many of which you can repeat for yourselves, and which may enable you to give something like a rational answer when "put to the question" in the manner suggested. I shall endeavour to be very elementary in my explanations. But I fear they will suffer much in this printed form from the absence of audible experiments and illustrations, and hence I must assume a slight acquaintance with some musical instrument. The reader is also expected to understand the four first rules of arithmetic, and, to a slight extent, vulgar fractions.

3. *How Sound is conveyed.*—Of course musical sounds have existed as long as there have been brains to receive the sensation

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* It is only necessary to mention my four papers on music, printed by the Royal Society in 1864 and 1874, and my translation of Helmholtz's work "On the Sensations of Tone as a Physiological Basis for the Theory of Music," with an additional Appendix, 160 pages long, written by myself, published in 1875. Also, since this lecture was delivered, my paper "On the Measurement and Settlement of Musical Pitch," in the *Journal of the Society of Arts*, 25 May, 1877.

of sound, and the causes of that sensation no doubt existed long before brains, just as now they exist where no brains, at least no human brains, are there to receive the sensation. Here is a toy, bought at a toyshop for threepence, called the Swiss Telegraph. You observe that it consists of two tin hoops (cylinders, about an inch long) with a piece of membrane (bladder or parchment) tied over one end of each, and a long string passing nearly through the centre of each membrane and prevented from slipping by a knot inside. I give one end to my assistant, requesting him to walk to such a distance as to make the string tight, without pulling the knot through, and to put the open end to his ear. I apply the open end of the other hoop to my mouth, and whisper a few words sharply. You hear he repeats them. He has heard them, though none else in the room did so most probably. Now this instrument is a kind of model of the process by which we hear. My whispering represented the air set in vibration, and impinging against the drumskin of the ear, the first membrane, which it made to vibrate. Then this vibration was carried on through the stretched string, representing the three curious little bones, called the hammer, the anvil, and the stirrup, inside the drum of the ear, and conveyed to the second membrane, representing the covering of the oval window of the internal ear, which is attached to the stirrup. The second membrane of the telegraph sets in motion the mass of air inside it, representing the watery liquid inside the internal ear, in which are immersed the many thousand ends of the nerve of hearing, and these must be considered to be represented by the drumskin of the ear of my assistant. This is how we hear in all cases. The difference in our sensations depends entirely on the different forms and velocities which the vibration of the air assumes, and which in this case were sufficient for my assistant to recognise the words I uttered. It is on this difference of form and velocity that depends the difference between music and noise, and between the musical sounds of various instruments and various human voices. As Sir Charles Wheatstone shewed, if instruments be played on a platform down in the cellar, and a stiff wire be attached to the platform and be led into a room at the top of the house, and then a board be placed on the wire, every note of the instrument is heard distinctly. Here is a simple illustration. I lay my watch on the table, face downwards, on a folded cloth, to prevent the communication of the vibration of its ticks to the table. Then I put one end of a poker on the back of the watch. I do not hear the tick. I apply my ear to the other end of the poker, and hear it very distinctly. Many an old woman has done the same kind of thing to hear if the kettle boils. Now, instead of my ear, I apply a drawing board to the end of the poker, and I and several others can hear the sound at the same time. And in such cases it is only the force, rapidity, and form of the vibration which could be conveyed. But the most striking instance of this transmission of vibration, conveying sound, occurs in Mr. Graham Bell's Telephone, the success of which was attested by Sir William Thompson at the Glasgow meeting of the British Association in 1876, as he had heard it himself. In this instrument the voice is made to act upon the armature of a magnet, which generates currents of electricity that pass through any number of miles of wire and produce audible vibrations in the armature of a magnet at the other end. And so exactly are these vibrations transmitted that words spoken at one end can be accurately distinguished at the other, and even the tone of voice of the speaker is recognised. Songs *with* words have thus been sent by telegraph, and on 13 Feb. 1877 the *Boston Daily Globe* published the first newspaper dispatch, of 110 lines long, sent by Telephone over a distance of 18 miles.* And yet the mode of conveyance is only the single motion of the single spring conveyed through a wire.

4. *Nature of Simple Tones.*—Now I begin with the simplest possible form of vibration, which is seldom met with except artificially, and which I shall exhibit in the way discovered by Helmholtz and first published in 1862. Is it not marvellous that for 2500 years in Europe, and much longer in China, people had been busy with music, and that it was not till 15 years ago that a man now living, and younger than myself, published his discovery of the real nature of musical sounds, the real basis on which the great superstructure of modern music had been raised by a mere rule of thumb observation without theoretical knowledge? Scarcely one of the numerous questions with which I began, admitted of a satisfactory answer before 1862. Here is a common "treble C" tuning fork. I strike it on a piece of lead covered with wash leather, to prevent tinkling. This sets the prongs into strong vibration, as you can hear by my bringing them against a piece of glass. They are actually vibrating, as nearly as possible, 512 times backwards and forwards in one second of time. I shall give you a notion how this was found out, before I have done; it was by no means easy. Of course it makes the air about it vibrate equally fast. But it sets so small a quantity of air in vibration out of the great mass in this room, that it probably does not cause the drumskins of your ears to vibrate and so you hear nothing. Now here before me are sundry jars, prune jars and others, with a little water at the bottom merely to bring the quantity of air within them to the proper shape and size to vibrate to different numbers of times in a second, which has been ascertained by previous trials like the following. I put my fork over the larger ones, and I get no result. The air in these will not readily vibrate so fast as 512 times in a second. But I bring it over a smaller one, and instantly a beautiful bright clear sound starts forth.† The air in the bottle is ready to vibrate at that pace,

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* And on 5 April, just after this lecture was delivered, an audible conversation was kept up for 2 hours through 43 miles of wire from Providence to Boston, U.S. America.

† The prongs of the tuning fork should be held duly horizontal, with either the flat side of one prong, or the thin edges of both prongs, over the mouth of the jar. If the prongs are oblique, either no sound or a poor sound will be produced. On rolling the end of the fork between the fingers the sound will alternately disappear and be reproduced, causing the effect of "beats."

and this mass of air when vibrating is sufficient to set so much of the general air of the room into vibration that you probably all hear it, although my apparatus is better suited for a small class. This phenomenon is called "resonance." Now observe, that certain forms and sizes of resonance chambers totally or partially extinguish certain vibrations and reinforce others. On this fact is based the whole construction of organ pipes, and keyed wind instruments; and in fact all phenomena of speech, our vowels especially, and the greater part of the singer's art. The immense field of application of this one principle obliges me to limit myself to this passing remark. The sound which you hear when a tuning folk is held over a proper resonance chamber, is called a *simple sound.* Each particle of air then vibrates exactly as a pendulum, with perfect continuity, and no jolts, decreasing in velocity till it stops, and then turning to go back, and so on. In a clock pendulum we count every swing and every swang (as De Morgan called it), meaning swing forward and swang backward. But in music we count each swing-swang as one vibration in England and Germany, though in France they count the swings and swangs separately. Thus, in the present case, the air in one second of time makes 512 swing-swangs or vibrations both ways, that is, 1024 simple swings each way, that is, simple vibrations, as the French call them, in one second. I shall always mean swing-swangs when I speak of vibrations.

5. *Nature of Compound Tones.*—Now I take another fork which makes half as many vibrations in a second, that is 256. Observe that it does not sound over the middle jar, but it produces another simple tone over the large jar. The tone is not so bright and clear as the former. It is somewhat "woolly." We call its note "middle C." Here is another fork which vibrates half as fast again, or 384 times in a second; it "speaks" well over the central jar, but not over the others. The note is called the "treble G,"—never mind why these names are given.

Now I will compound the treble with the middle C. I strike both and hold them over the proper jars at the same time; and, that you may hear the difference of effect, I sometimes remove one and sometimes the other. When I remove the middle C the note, as we say (never mind why), "runs up an octave"; but when I remove the treble C, the pitch does not seem to alter, but only the kind of tone, or "quality," as we call it, alters. When both are sounding, after the first second, you cannot tell that two are sounding; it seems to be one single musical sound and not two. But on hearing first both treble and middle C together, and then the middle C by itself, we find that the quality of tone of the combination is much brighter and pleasanter than that of the middle C alone. Now, again I strike the middle C fork alone, and hold it over the treble C jar. The sound is not altogether extinguished, as that of the treble C was when held over the middle C jar. Listen, is not the sound a weak treble C? I sound the treble C but hold it at a distance from the jar, to make the sound weaker, and then, in rapid alternation, get the resonance, first from this fork and then from the other. Those who are near enough to hear will, I think, acknowledge the sound to be the same.*

Listen while I pass rapidly with the middle C fork from one jar to the other, and observe how the note changes. How is this? Helmholtz's answer is, that the sound produced by the middle C fork is really compound, that it really contains both middle C and treble C, and in fact several other sounds, but that the resonance jars extinguish some of the sounds in both cases, and bring out only a simple middle C in one, and a simple treble C in the other. Let us try. Here again I sound both middle and treble C together, then I take both from the jars, and bring the end of the middle C fork on the table, by which means you are able to hear the whole sound of the fork. Now, I first hold the middle C fork over the jar alone, and then bring it on to the table. Does not the table sound (as I may call it) much more resemble the double-jar sound than the single-jar sound? Certainly, but it is not the same, it is not so bright and pleasant, because there are other sounds also existing on the fork which impair its beauty.

6. *Nature of Musical Tones.*—Now this is a faint indication of the great discovery of Helmholtz, respecting the nature of musical tones. All the tones we hear in musical instruments are more or less compound, those of the flute and the deep-toned wide-stopped pipes of organs are about the simplest. But for all others, the ear hears them as if they were made up of simple tones, just in the way that I have shewn you; only, instead of 2, there are 6, or 8, or even 16 or 20, sounding together. Suppose the "prime" or slowest of these "partials," (as we may call them, because they are the parts of the whole "compound tone" or " note,") to make 1 vibration in a certain fraction of a second, then the next quickest makes 2 vibrations in the same time, and is called the Octave of the prime; the next quickest makes 3 vibrations in the same time, and is called in musical language the Octave-Fifth or Twelfth of the prime; the next quickest makes 4 vibrations in the same time, and is called the double Octave of the prime; the next quickest makes 5 vibrations in the same time, and is called the double-Octave-major-Third or Seventeenth of the prime; the next quickest makes 6 vibrations in the same time, and is called the double-Octave-Fifth, or Octave-Twelfth, or Nineteenth of the prime; and so on. The rates of the vibrations of the different partials are always found by multiplying the rate of the prime by the numbers 1, 2, 3, 4, 5, 6, and so on. And upon this fact, as we shall see, all musical scales, and hence all music, has depended, and must depend, from all time to all time. And yet the world did not know of it till 1862!

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* These forks had been made very accurately for me by Mr. Greaves (now Valentine and Carr), of 76, Milton Street, Sheffield, and cost 1*s*. 6*d*. each. But a middle and treble C fork, by no means so accurately tuned, which can be purchased at any music shop, will answer the purpose. The experiments with forks and resonant jars should be tried, or the reader will fail to comprehend the most elementary fact of music.

Now observe that all these partials are simple, that is, each of them causes the air to vibrate like a pendulum. But when we compound them, the vibration is by no means simple, but its form depends on the number and relative loudness of the different partials. This phenomenon has been much studied, but I cannot now enter upon the details and methods of study. I can merely give an indication of the results. You have heard simple tones which consist of primes alone. Here (playing some notes on the concertina, for which an harmonium may be substituted, but only single notes should be used) are very compound tones, with large numbers of the quicker partials, the roughening effect of which you will soon better appreciate. The notes on the piano have about 6 partials only, and, if they could but be sustained, would be very fine, as you may hear in the soft notes of the French horn. By striking a low note strongly on *one* string of the piano (use the piano pedal therefore), and, pressing the finger down in order to keep the damper raised while the sound dies off, you will hear the effect of the slower partials dying off one after another, making the note apparently run to the Octave, the Fifth above it, and so on. On the clarinet, the peculiar character of tone is due to the absence of the evenly numbered partials, as the second, fourth, sixth, &c. If you twang a harp or guitar string by pinching it exactly at the middle (which is against all rules) you produce the same effect. The oboe, on the other hand, gives both the odd and even numbered partials, and has many of them, and so have all reed instruments, including the human voice, all conical instruments like the trumpet, all bowed instruments like the violin, &c. But the relative loudness of these partials proceeds by very different laws in these instruments, and hence gives them their different characters or qualities of tone, and, as a general rule, the loudness of the partial diminishes at the same time as the number of its pitch increases; but not at the same rate, generally much faster, on the violin as the square of its number, that is the second, third, and fourth partials have only one-fourth, one-ninth, and one-sixteenth the loudness of the prime partial. As to the form of the vibration in the case of a compound tone with six partials of this kind, it may be described as jumping up half-a-dozen stairs at once, and then stepping down one stair at a time, and repeating this operation constantly; so that, though the jumping up (or swing) occupies a much less time than the stepping down (or swang), yet the whole time of doing both, that is, of the swing-swang or vibration, is the same for each operation. The time then of the swing-swang of the whole compound vibration is the same as that of the swing-swang of the simple prime.

7. *Distinction between Partial Tones and Harmonics.*—Before passing on to the next point, I would call your attention to the difference between a partial tone and an harmonic. A partial tone is always perfectly simple, and is also always part of a compound tone, from which it can be isolated only by the contrivance of resonance chambers like these jars. Such chambers when adapted for placing close to the ear are called "resonators." It

was by the help of resonators that Helmholtz made all his discoveries. But harmonics are all of them compound tones. Those tones you will hear while the sound of the piano-string dies away, are harmonics. You first hear the whole compound tone, having say 6 partials. You next hear only the 5 partials above the prime, that is, only the 2nd, 3rd, 4th, 5th, and 6th. Then only the 3rd, 4th, 5th, and 6th, and so on, and these peculiar combinations cannot be easily reproduced in any other way. Again, when you touch a violin or harp string lightly and exactly in the middle, and bow or pluck it, you produce only the 2nd, 4th, 6th, &c. partials, which by their composition generate that well-known "flageolet tone," as the Germans call it. If you have access to a flat piano, and measure one of the strings accurately, placing a mark to shew you exactly where the middle, one-third, and one-fourth points are, and then touch the string lightly (with a feather, is best), you will on plucking the string, after raising the damper or on striking it with the key (after putting down the piano pedal so as to strike one string only), get out three harmonics, the Octave (having the 2nd, 4th, and 6th partial of the full string), the Twelfth (having the 3rd and 6th), and the double Octave (having the 4th only, or the 4th and 8th on the plucked string.) These are therefore none of them really partials, because they are not simple, but they serve to represent the partials to our minds, because they are compounds of the quicker and fainter partials, and it is the 2nd, 3rd, and 4th partials respectively which most affect the ear. See also the artificial harmonics produced by independent strings, or reeds, or pipes, as defined in art. 13.

8. *On the Nature of Musical Pitch.*—Now, to attend to pitch. Here is a common piece of rather smooth even twine. I have tied one end to the leg of the table, and the other to the back of my chair, on which I sit to render it firm. I stretch the string slightly and pluck it. There is somewhat of a musical sound, and you can see the extent of the vibration very well, though special appliances are necessary to shew you the really beautiful form of that vibration. Now I push my chair a very little further. I of course slightly lengthen the string by so doing, but I stretch it so much tighter at the same time that the effect of the lengthening is inappreciable. I pluck it again, and you perceive that it vibrates much more quickly, and has a more musical character of tone. I must be careful not to stretch it too tight or it will break, as you know pianoforte strings and harp strings are liable to do when you screw them too tight. But you see sufficiently that tightening quickens vibration and slackening retards it. The effect on the ear when we quicken vibration is called "sharpening," and the "pitch" of the note is said to be "raised." By "pitch" is best understood the number of vibrations made in a second, although this is not easily found out—indeed, before 1834, when Scheibler invented his tonometer, it could be only roughly determined at best by a mathematical formula, with which I need not trouble you, depending on the length of a string, its weight per inch of length, and the weight by which it was stretched. By a note of

a higher pitch we then mean, a note making more vibrations in a second. The Greek and Latin words for raising and lowering pitch, or sharpening and flattening, simply meant tightening and slackening, as the string was always thought of. But now, observe, without altering the "tension," or stretching of the string, I grasp it between my finger and thumb nail so as to limit its length, and pluck it. The tone immediately sharpens, and I can get any number of different tones out of the same string. One can almost play a tune upon it.* Now we know mathematically, and it was known to the Greeks and Arabs, that if the number of vibrations of the full string is known, and this number is multiplied by the number of inches in the whole string and then divided by the number of inches in the part stopped off, the result is the number of vibrations of that portion. Thus if the whole length of the string is 48 inches, and the part stopped off be 32 inches, then we multiply the number of vibrations of the whole string by 48 and divide by 32, or what is the same thing (as these numbers have a common measure, 16), multiply by 3 and divide by 2, to find the number of vibrations of the piece stopped off. Hence, although we might never know the absolute number of vibrations, we could determine the relative number. Thus the number due to 32 inches would be $\frac{32}{36}$, or $\frac{8}{9}$ of the number due to 36 inches. In this way it was possible to determine the actual relative pitches of the notes used in singing. The singer sang a note, and the string was stopped where it gave what seemed to be the same note. Then the singer sang another, and the corresponding length of the string was again determined, and so on till all the notes of the scale were sounded. Then the mathematician measured off the distances, and calculated the relative pitch. In this way Euclid, 300 years before Christ, laid down the relative pitch of the notes of the Greek scale, so that we know accurately what the Greek scale was, and I have here an instrument tuned in that way, to which I shall have to draw your attention afterwards.

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* Drive two big nails into a board 48 inches apart, and stretch the string as tightly as you can between them. Paste a piece of paper on the bottom, on which are marked the inches from a rule. Then set off the following distances on the scale with these letters against them:—

D	$42\frac{5}{8}$	inches.	*d*	$21\frac{3}{8}$	inches.	*d′*	$10\frac{5}{8}$	inches.
E_1	$38\frac{3}{8}$	„	e_1	$19\frac{2}{8}$	„	e_1'	$9\frac{5}{8}$	„
F	36	„	*f*	18	„	*f′*	9	„
G	32	„	*g*	16	„	*g′*	8	„
A_1	$28\frac{6}{8}$	„	a_1	$14\frac{3}{8}$	„	a_1'	$7\frac{2}{8}$	„
B_1	$25\frac{5}{8}$	„	b_1	$12\frac{7}{8}$	„	b_1'	$6\frac{3}{8}$	„
C	24	„	*c′*	12	„	*c″*	6	„

By gripping the string over these letters any tune in a major scale can be played within the compass of three octaves, assuming the full length of the string to give the key note. The first set of numbers are $\frac{8}{9}$, $\frac{4}{5}$, $\frac{3}{4}$, $\frac{2}{3}$, $\frac{3}{5}$, $\frac{8}{15}$, and $\frac{1}{2}$ the length of the string (to the nearest eighth of an inch), the second set half, and the third set a quarter of these, to the nearest eighth of an inch.

Now melody depends on relative pitch only. A woman or boy can sing the same tune as a man, and they can also begin on any note which suits their voices best, and still the tune is the same; though in a great many other points the effect is different. Listen while I play the opening bars of *God Save the Queen*, at different pitches and in different keys.

9. *Beats of Simple Tones.*—We have now a very important fact to ascertain, which is mainly instrumental in distinguishing music from noise. I hope you will have observed that I have studiously refrained from attempting to do so hitherto, and may have wondered why. This fact also depends on absolute pitch. I have here three beautiful tuning forks mounted on resonance boxes, which are protected from the table by pieces of indian rubber tubing. They were made with his usual perfect accuracy by Herr Georg Appunn, of Hanau, near Frankfurt-on-the-Main, in Germany, and may be relied upon to give exactly 435, 440, and 444 vibrations in a second, which are the theoretical pitches of the A string of the violin in France, in Germany, and at the Society of Arts, England, though, owing to inefficiency of the means of measuring in 1859 and 1860 in France and England, the actual French "Diapason Normal," as it is called (I have myself measured an authentic copy presented by the French commission to Messrs. Broadwood in 1859), really makes 439 vibrations in a second, and the fork sold by Messrs. Cramers as the A fork of the Society of Arts makes 452, though the Society's own fork makes 449·4 vibrations.* And I have here also a common French A fork which makes 438 vibrations in a second very accurately. Now I strike the 440 fork with my piece of lead covered with wash-leather, and, striking the 438 fork, I hold it over the mouth of the resonance box of the former. Immediately, you find that the continuous even sound of the first fork is broken up into beats of sound as it were, separated by distinct silences. And if you look at your watches and count (it is interesting to do so in all these cases), you will find that the greater

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* For full information respecting the measurement of pitch and the actual pitch of standard forks, see my two articles on Tonometry, in the *Athenæum* for 2nd and 30th December, 1876, pp. 731 and 893, and the *Journal of the Society of Arts* containing a paper which was read by me on 23 May, 1877, "On the Measurement and Settlement of Musical Pitch." It is interesting to know that Handel's own A fork gave $426\frac{3}{5}$ vibrations, being rather more than a semitone flatter than that in actual use in England, and that Messrs. Broadwoods' three pitches for C, are —low 523, medium 535, and high 545, almost the highest known. Handel's A made his C $510\frac{1}{10}$, the actual French C has 522, and the C of the Society of Arts, as sold by Cramer, has 538, instead of 528 as was intended. The Society's own C fork makes $539\frac{1}{3}$. It is as well to remember that at the various pitches of C here mentioned a difference of 3 vibrations corresponds to a difference of one-tenth of an equal semitone in pitch; and that at the various pitches of A, a difference of 5 vibrations corresponds to a difference of two-tenths of an equal semitone in pitch.

loudnesses and the silences recur at intervals of half-seconds. Now 440 less 438 is 2; the number of "beats," as these loudnesses are called, in one second is therefore the difference of the number of vibrations or of the pitches of the two simple beating tones. Now I stop the 440 fork and strike the 435, and then apply the 438 as before. The beats are now faster, being 3 in a second, the difference of the numbers as before. Similarly, observe that 440 and 444 give 4, 435 and 440 give 5, 438 and 444 give 6, and 435 and 444 give 9 in a second; so that, if these pitches are correct, the rule is established that the number of beats is the difference of the pitches. But I must tell you that the pitch of all these forks was determined by the numbers of beats after assuming this law to be true, and hence it was necessary to establish the law otherwise, as has been done especially by means of Helmholtz's instrument called "the double siren." This fact, however, I must ask you to take for granted.

But this is not all that is to be said of beats. Two simple tones will not beat if their relative pitch, or the fraction that the higher pitch is of the lower, exceeds a certain amount, and that amount again depends on the absolute pitch of the deeper note. In the deep bass, this fraction must not exceed $\frac{5}{4}$, indicating that the lower note must not be more than a major Third above the upper one. Through the upper part of the bass and most of the treble, the fraction must not exceed $\frac{6}{5}$, indicating a minor Third, and higher up it is still less. But the beats are not particularly offensive when the notes are so far apart. I here sound an A fork and a C fork, both wrongly tuned, so that the relative pitch is less than $\frac{6}{5}$, and you will hardly notice the beats.* Even these two forks, the G and the various A's, hardly beat distinctly. It is only when the relative pitch of the notes is nearly $\frac{16}{15}$ or less that the beats become really annoying. I will sound some pairs of forks within or beyond that distance, gradually increasing the interval from a pair which gives 4 to a pair which gives 36 beats in a second. You see in all cases that the beats destroy the musical character of the sound. Tones are musical when every one of their vibrations is completed in exactly the same time, such as simple tones, or compound tones made up of such simple tones as are not near enough to beat disagreeably. But when the vibrations are not uniform, or when beats arise, musical tone proper ceases, and noise begins. But there are noises and noises, and some of them naturally produced while changing one musical tone into another (in the discords from "passing notes" which are so well known), rather add to the enjoyment of the coming musical effect ("resolution"). Others, arising from the nature of compound tones, are very rapid indeed, and simply characterise combinations. Others, due to our artificial method of tuning, we have learned to endure in our orchestras, and harmoniums, which, as we shall see directly, are with *malice prepense* stuffed full of noises that we try to overhear and leave unnoticed.

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* When both are held up close to the *same* ear, the beats are very offensive. If one is held to one ear and one to the other, the beats are not heard at all. This depends upon the different relative position of the nerves affected.

10. *Beats of Compound Tones.*—So far I have dealt only with simple tones, and have shewn that the number of beats depends on the difference of the pitch, and may be very small indeed, as in these two forks, where they are less than one in a second, and where the beat seems like a continuous sound just momentarily stopped out at regular intervals of time. All beats really occur between simple tones, and produce intermittences of sound separated by total silence so far as the beating tones themselves are concerned. But when two compound tones are sounded together, there will be several which are beyond the beating distance, and which go on sounding continuously, and thus obliterate the silence and disguise the beat. I here sound a compound middle C, and with it a treble C which is not a true Octave to it. You will, if you listen attentively, perceive a decided beat, but there are many low and high partial tones in the compound note which go on quite unconcernedly, and you may at first only think that there is something queer about the sound. Afterwards, you will find that the second partial of the compound tone has been challenged to single combat by the fork, and notice the clinking of their swords! I will now play two compound tones nearly alike, differing by what is called a comma (not quite the fifth part of a semitone, relative pitch $\frac{81}{80}$), and you will observe the extreme difference of effect from beats of about the same interval on forks, and the altering of the number of the beats as I take the interval higher. It is by means of these beats that we are able to count vibrations. In the old imperfect way already mentioned we find a fork or harmonium reed giving a note of nearly 256 vibrations, but we do not know exactly how many. Then we form in succession, with immense care, a series of notes each beating 4 times in a second with its two neighbours; thus we get forks with 4, 8, 12, 16 and more vibrations than the first fork, without knowing exactly what the actual numbers are. We go on for 62 forks till we get to a fork having 248 more vibrations than the first. Suppose we find that this last fork beats twice in a second with the starting note, that is, its prime beats with the Octave of the starting note twice in a second. But is it sharp or flat? To determine, we tune a 63rd fork, which would have 252 more vibrations than the original. Suppose we find that it also beats 2 in a second, and we know that the lower one has 248 and the upper one 252 beats more than the lowest. Hence the lowest pitch increased by 248, and 2, the beats heard, (or 252 less 2, the beats heard,) gives twice the lowest pitch. The lower note must then make 250 vibrations in a second, and the pitch of all the notes is accurately ascertained.*

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* A beautiful instrument with 65 reeds, making from 256 to 512 vibrations in a second, and constructed in this way, was exhibited in the

11. *Nature of Differential Tones.*—It is necessary to hurry on, and hence I can only say a few words about another curious and important point respecting simple tones. If any two of them be sounded *loudly* together, they generate a third tone, called in Germany, where it was discovered, by Sorge in 1740, a "combinational tone." It has been variously called in England the "grave harmonic," and the "resultant tone." I shall term that particular kind of combinational tone which I intend to shew you the "differential," because its pitch is the difference of the pitches of the generating tones. Here are two flageolet-whistles or fifes. On them I and my assistant can blow two very high notes indeed, which are practically simple tones. Never mind what their names are, they are sure not to be what they profess to be.* Now, observe my assistant blows first, and then I blow a different tone, somewhere about a semitone lower; and, the instant I blow, remark, first, that there are no beats, because the notes are so high that they are on the verge of the beating distance, but there is certainly an unpleasant sort of scream. Experimenters must submit to annoyances. I have listened for minutes together with profound attention to the most disagreeable masses of beats possible, enough to thump one out of the room, for the purpose of ascertaining that they occur according to rule. So I must ask you to put up with this scream. But remark, secondly, that quite independently of this, there is a very deep tone, about four octaves lower than what I am playing, about the pitch of the violin G or lower. This is the "differential," which is a simple tone. Next I will first sound the note which produces this tone, and then change to a lower tone. Observe that in this case the difference of the pitch is greater, and hence the differential is much higher, and you will distinctly hear it jump up and down again. These differentials vibrate as many times in a second as the notes would beat, and it is, curiously enough, possible at times to hear both the beats and the differential tone. I can, however, shew this only with compound tones, where the beats and the differential must be considered as produced by their primes. And I cannot make them audible to all the room at once. I play high notes, having a relative pitch of about a semitone or less ($\frac{16}{15}$ to $\frac{25}{24}$), on my concertina. By placing the ear against the bellows you will hear the sharp rattle of the beat, and the low booming differential, something like a threshing machine two or three fields off. When you have once got used to hear these differentials, they are often painfully conspicuous, especially on the harmonium, where they add greatly to its disagreeable effect. On account of the

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Loan Exhibition of 1876, by Herr Georg Appunn, and has been purchased by the South Kensington Museum, and another copy by King's College Physical Laboratory. It is with this instrument that I made the measurements given in the two articles in the *Athenæum* and the paper read before the *Society of Arts*, mentioned on p. 12, note.

[*] They professed to be a G, 4 octaves higher than the lowest G of the violin, and the F sharp immediately below that G.

short time that notes last on a piano, and the rapid diminution of their force during that time, these differentials are but little heard on that instrument.

12. *The Basis of Music, and the kind of Instruments best suited for its Demonstration.*—We have now got the real basis of music. We know the difference between simple and compound tones, or the *law of compounds*. We know the *law of beats*, and the *law of differentials*, by which the partials of these compounds interfere with each other; and we know that to get agreeable sounds we must steer clear of conspicuous beats (that is, comparatively slow beats of comparatively loud tones), and employ tones which are moderately compound. Now, what are we to do? The sustained tones of the human voice are the real compound tones with which we must deal primarily. The unsustained tones of the piano, harp, guitar, lyre, and plucked strings in general, cannot exhibit the full effects of music. The violins and orchestral instruments require so much practice that we must disregard them at first, to keep them for special studies afterwards. The voice we have always with us. But it is rather unmanageable and requires guidance. We cannot always be sure of getting the note we want, and still less can we be sure that the voice will not slide out of difficulties and leave the experimenter stranded. Again, a voice can only produce one note at a time. We cannot call up a choir at will. For our experiments, then, we want an instrument with sustained tones, having very numerous partials, which can be tuned easily, which will keep in tune well, and on which many notes can be played at a time. The harmonium seems to answer perfectly, and an ordinary harmonium may be retuned so as to answer very well for many purposes,* but we ought to have many more notes to the Octave, as in Mr. Bosanquet's and Mr. Colin Brown's harmoniums. It so happened that as a boy I

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* I have had one tuned justly by the following rule, where F^1 is placed upon the F sharp key, and the other notes remain in their usual places: Tune the following seven major chords perfect without beats (art. 25.): FA$_1$C, CE$_1$G, GB$_1$D, D^1*fl*FA^1*fl*, A^1*fl*CE^1*fl*, E^1*fl*GB^1*fl*, B^1*fl*DF^1. This allows of playing all kinds of perfect major and minor chords, and gives the major keys of C, A^1*fl* and E^1*fl*, and the minor key of C, which suffice for merely experimental purposes. (See the Notation explained in art. 22.) Arranged as in the contracted Table of Harmonics of B*fl*, in art. 25, the the scheme of this harmonium is

F^1
B^1*fl* D
E^1*fl* G B$_1$
A^1*fl* C E$_1$
D^1*fl* F A$_1$

from which, after reading articles 23 to 27, the whole power and capabilities of the instrument can be shewn. It contains the modulation from A^1*fl* major into its dominant E^1*fl*, and conversely from E^1*fl* major into its subdominant A^1*fl* major, and also into its relative minor C. In E^1*fl* major it has the two forms of the Second F^1 and F, which can be contrasted. In C major the grave form of the Second or D$_1$ is absent.

learned to play on Wheatstone's English concertina, which is more portable and cheaper than the harmonium, and having 14 keys to the Octave, gives greater facilities for experimental tuning. On these instruments I made all my experiments long before Mr. Bosanquet and Mr. Colin Brown had invented their harmoniums. I have four of these instruments here, all tuned on different principles,* and I will shortly play a few bars on each to make you hear the effect. I should add that the quality of tone in all reed instruments is rendered so harsh by the great number of partials which are all inclined to beat with each other, that what sounds decent with them would be lovely with the human voice.

13. *Application of the Basis of Music to the Formation of Scales.*—Now what do we want to do? The natural spontaneous song of untaught man replies: "We want a number of perfectly definite and distinctly different musical tones, which, when sounded in succession, will call our feelings into action." To which cultivation adds: "and which have a good effect when sounded together." But how are we to select those tones? Here there seems at first to be no kind of principle at work, for different nations have chosen very differently, and our own present choice seems remarkably strange, and is, at any rate, very modern and extremely artificial. But a little examination shews that the earliest Chinese and Gaelic, and the later Greek and Arabic, had a decided principle, although the earlier Greeks had a different one, which is not so easy to appreciate. In the following exposition, I shall, for brevity, take only this principle, and the chief modern scales; and, in order to under-

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(Those who prefer it, could have D_1 tuned on the key which is usually D^1*fl*, but then the dominant and subdominant modulations will be absent.) It has the modulation from C major into C minor. Music would, of course, have to be arranged for it. The whole of the "harmonics" of C (art. 13) can be readily shewn, and by driving brass-headed nails at proper distances into a lath, can be made to speak at once, or the first set separately, 1C, $2c$, $3g$, $4c'$, $5e_1'$, $6g'$, $8c''$, $9d''$, $10e_1''$, $12g''$, $15b_1''$, $16c'''$. This and the concertina are the cheapest experimental instruments that can be obtained. Any harmonium tuner can tune an ordinary harmonium in this way. My own was tuned in two hours under my directions in my own presence.

* One of these was in the Greek Tuning (art. 19), another in the Old Tuning (art. 30), a third in the New Tuning (art. 31), and a fourth in Just Tuning, having the eight perfect major chords (art. 25), B*fl* D_1 F, FA_1C, CE_1G, GB_1D, DF_1*sh* A; A_1 C_2*sh* E_1, E_1 G_2*sh* B_1, B_1 D_2*sh* F_1*sh*. If these are arranged as in the Contracted Table of Harmonics of B*fl* in art. 25, it will be found to contain 14 out of the 16 notes there written, the omitted notes being E at the top, and F_2*sh* on the right at the bottom. This gives a very great deal of power, and allows of the arrangement of numerous airs, so as to shew the nature of Just Intonation, besides giving examples of all kinds of chords and modulations. It is a very cheap instrument, and Mr. Saunders, of Lachenal's Concertina Factory, 4, Little James Street, Gray's Inn Road, W.C., has my instructions for tuning it in this way.

stand their formation, I examine the effect of the combination of two compound tones together. This examination is made for the purpose of learning, first what the relative pitches of two or more notes must be which will sound well when made to speak at the same time; secondly, what notes when sounded in succession will be related to each other by means of common partial tones; and, thirdly, what is the limit of the endurable amount of noise and roughness generated by slight alterations or modifications of these relative pitches.

Take what we will call a "fundamental" tone,—on the concertina the violin G, which vibrates on this instrument about 200 times in a second, and hence vibrates once in the 200th part of a second. Its prime therefore vibrates as fast, and the other partials, 2, 3, 4, 5, 6, and so on, times as fast. There are many more than six partials of course. But these six will have the pitches known as *g*, *g′*, *d″*, *g″*, *b″*, *d‴*, or in musical notation as shewn in the following diagram, all being in this case simple tones.* Now tune the compound tones which are "harmonics" of the deepest note, or *g*; that is, which are compound tones having these simple partial tones of *g* as their primes. In this sense, the harmonic of a fundamental is a compound tone erected on one of the partials of the fundamental as a prime. These harmonics are of the nature of those obtained by dividing a pianoforte string, as explained towards the end of section 7. They are not the same, however, because they are equally powerful with the original note, being, in fact, all original notes. Listen while I sound them on the concertina. You cannot hear them on the piano, because the notes are there purposely varied. The 1, 2, and 4 are right on the piano, and the 3 and 6 are fairly right, but the 5 is very wrong.

Now, what will be the pitches of all the partials in these six notes, *g g′ d″ g″ b″ d‴*, taking only six of each, by way of illustration, but bearing in mind that others of a still higher pitch may, and in many cases do, exist? Write them as in the following diagram, where each number must be supposed to be multiplied by 200, as shewn in the last column, and where the numbers in parentheses in the first column indicate other partials in the fundamental out of the many which may be supposed to exist. On some reed tones I have been able to shew that at least 19 could be made effective.

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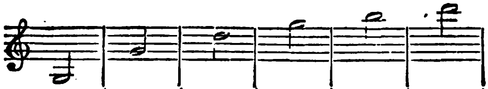
* In this common mode of denoting pitch by letters, the whole set of musical notes are divided into Octaves, beginning with C. The capitals C D E F G A B are the notes beginning with C on the second leger line below the bass staff, that is, the lowest note on the violoncello; the small letters *c d e f g a b* denote the next Octave above, beginning with *c* in the second space of the bass staff, that is, the lowest note of the viola, the *g* being the lowest note on the violin; the once accented letters *c′ d′ e′ f′ g′ a′ b′* denote the next Octave, beginning with "middle *c′*" on the leger line between the bass and treble staves; and each successive Octave adds an accent, but c^{iv} and c^{v} are used for *c⁗* and *c′⁗*, &c.

DIAGRAM OF THE HARMONICS OF G WITH THEIR PARTIALS.



1 g	2 g'	3 d''	4 g''	5 b''	6 d'''	Names.	Vibrations.
1						g	200
2	2					g'	400
3	...	3				d''	600
4	4	...	4			g''	800
5	...	...	...	5		b''	1000
6	6	6	...	...	6	d'''	1200
(8)	8	...	8	...	...	g'''	1600
(9)	...	9	...	...	...	a'''	1800
(10)	10	...	...	10	...	b'''	2000
(12)	12	12	12	...	12	d^{iv}	2400
(15)		15	...	15	...	f_1^{iv} sharp	3000
(16)		...	16	...	...	g^{iv}	3200
(18)		18	...	...	18	a^{iv}	3600
(20)			20	20	...	b^{iv}	4000
(24)			24	...	24	d^{v}	4800
(25)				25	...	d_2^{v} sharp	5000
(30)				30	30	f_1^{v} sharp	6000
(36)					36	a^{v}	7200

These numbers shew the relative pitch of all the partials involved, and the musical names and full pitch of the partials are given on the right. I will speak of these compound tones as the *notes* 1, 2, 3, 4, 5, 6, each of which involves all the partials written in the column which it heads, and I will distinguish the latter as the *partials* 1, 2, 15, &c.

14. *Octaves.*—Now suppose we first play the note 1, and then the note 2 (sounded), it is quite clear that there is much in the second note the same as in the first. The partials 2, 4, 6 are in both, but the latter has also the partials 8, 10, 12. The resemblance between the two sounds is so great that for a very long time they have been called by the same name, and even the best ears are sometimes deceived in appreciating the difference. Also, naturally, male and female, or men's and boys' voices sing Octaves to each other, when they endeavour to sing in unison. And as men and boys certainly must sing together, in all countries we find the Octave not only used everywhere in succession, after the lower note, but sounded at the same time with it. Observe the difference between *g* alone, and *g* and *g'* together, and that the latter sounds more like a fresh note of the same pitch with a new quality of tone than like two separate notes.

15. *Twelfths.*—Now sound notes 1 and 3 after each other. They are so wide apart that the relation is not well heard, although note 3 has two partials 3 and 6 (its first and second) identical with two of note 1 (its third and sixth, which, being

high and comparatively weak, do not strike the ear so much). If I sound them together, the effect is rather strange, because the reinforced partials lie so high, and because of the additional higher partials. Few voices get notes so far apart within their compass, and though the Greeks recognised that such notes were consonant, they never sounded them together.

16. *Fifths.*—Now sound the notes 2 and 3 after each other. Here, there is a decided relation felt, and it seems due to the coincidence of the second partial of note 3 with the third partial of note 2, being the partial 6. Partial 12 is also the same in both. Every nation has recognised this relationship. It has lately been found by accurate experiment* that human ears recognise the relationship of these two tones, notes 2 and 3, better than any other relationship between two consecutive notes. Given any note taken as 2, a practised voice will sing its 3, with perfect accuracy, even better than its Octave or the Unison. But capital as it is, and clear and bright as is the combination *g′* and *d″*, (which is called a Fifth), its effect is hard and strong, and musicians, though constantly using this interval in conjunction with others, have forbidden its employment between the same parts in two following chords (this is known as the prohibition of consecutive Fifths). The Greeks do not seem to have used it at all in combination, but it became the principle of the selection of notes for the scale among them, the Chinese, the Arabs, and the Gaels. If we play all the three notes 1, 2, 3 together, the effect is very hard indeed. This is done occasionally on some stops of the organ, but it cannot be long endured. I will play you a bar or two to shew you the effect. It cannot be done except on specially tuned instruments.†

17. *Fourths.*—The next succession and combination to be tried is that of notes 3 and 4. Here the relation is by no means close, and it was probably obtained by the Fifth and Octave, on playing the notes 2, 3 and 4 in succession. The combination of notes 3 and 4, when played together so high as *d″ g″*, or even an Octave lower, is not at all unpleasing, but when it is taken low down in

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* By Dr. Wilhelm Preyer, of Jena, see his pamphlet on "The Limits of the Perception of Musical Tone," (*Ueber die Grenzen der Tonwahrnehmung,*) Jena, 1876. An account and rearrangement of this Paper was given by me to the Musical Association in November, 1876, and will be printed as the first paper in its Transactions for 1876-7.

† It cannot be done in part music without going out of the key. In the example given (the first six bars of *God Save the Queen*), while the two lowest notes of each chord forming Octaves were in the key of C, the highest note, or Twelfth, was in the key of G. This is one reason why consecutive Fifths are forbidden. But when all three notes are sounded in each chord, it was impossible to distinguish three parts. The effect was that of single notes of a very hard quality. This is another reason for the prohibition. For a succession of Thirds and Sixths, varying between major and minor, the key is preserved, there is no hardness, and consecutive Thirds and Sixths are in constant use. There is no resemblance to single notes. Hence they are not forbidden.

the base, the beats from partials 15 and 16 become very offensive, and hence in older music, which had reference to male voices in especial, this combination is treated as discordant.

18. *Chinese and Gaelic Scales of Five Notes, forming Fifths.*—Now here the early musicians stopped. They never sang except in melody, in unison, or in Octaves.* The instruments gave the same notes as the voice, and served only to steady it. The most ancient plan went on by Fifths or notes 2 and 3 only, and then brought the notes down by an Octave. Thus from the four Fifths, *fc'*, *c'g'*, *g'd''*, *d''a''*, they obtained five notes, *f c' g' d'' a''*, which, when brought into the same Octave with the first note repeated, gave *c' d' f' g' a' c''*, which hardly form a scale at all to our ears. You find that the semitones are all wanting, and that there are two intervals, *d'f'* and *a'c''*, which form minor Thirds. With this scale the singer began wherever he liked, and ended whereever he liked. Doubtless, there were many refinements which cannot now be appreciated by us. You shall judge of the effect for yourselves. I will first place a Chinese piece given by Barrow, and then the Scotch *Braes of Balquhidder*,† which both run on these five notes only; and as the five black notes on the piano represent them, (being in fact *f sharp, c sharp, g sharp, d sharp*, and *a sharp*,) you see how it comes to pass that you can play Scotch tunes on the black keys, by taking a hop skip and a jump now and then.

19. *Greek Scale of Seven Notes forming Fifths.*—But the Greeks did not rest content with these five notes. They went on and formed six Fifths, *fc'*, *c'g'*, *g'd''*, *d''a''*, *a''e'''*, *e'''b'''*, or seven notes, *f c' g' d'' a'' e''' b'''*, which, when reduced to one Octave, with the first note repeated, gave *c' d' e' f' g' a' b' c''*, the foundation of our own musical scale, and of all modern music. But they treated this in a variety of ways which I have no time to notice, and which were totally different from our own. You may, however, take the following *Hymn to Calliope*, as interpreted by Mr. W. Chappell (History of Music, p. 170), as an example. Now, if they had our scale, why did they not play harmony? I did not say they had our scale, but merely that their scale was the foundation of ours. Why they did not play harmony, you may judge for yourselves. Here are a few bars of *God Save the Queen*, harmonized according to modern rules, using the Greek notes such as Euclid gave us. They do not sound very inviting. What makes them so harsh? I must go to the next two pairs of relations, notes 4 and 5, and notes 5 and 6, before I can reply at all.

20. *Major and Minor Thirds.*—Observe the columns of notes 4 and 5. It will be seen that they have no partials in common

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* This is still the case among the Chinese, Indians, and Arabs, though their scale is much more complicated even than the Greek, and totally impossible to represent in ordinary musical notation, or play on European instruments.

† The notes will be found in my translation of Helmholtz on the Sensations of Tone, p. 412; in the Scotch air I omitted one note due to its modern form.

till we come to partial 20, which is the fifth partial of the deeper and fourth of the higher note. This connection is very remote, and it is notorious that singers have considerable difficulty in striking a true major Third, (as this interval is called,) and can easily put up with some change in it so far as melody is concerned. Again, observe that, in playing the two notes together, the partials 15 and 16 will beat, and also partials 24 and 25, very distinctly, if the notes are in the bass. You will detect a roughness even when I play as high as *g b*, (where the lowest note is the violin *g*,) which is greatly diminished in the higher Octaves. Hence, it was a long while before musicians could discover it, or make up their mind to use it. Again, take the notes 5 and 6, forming what is called a minor Third. Here again there are no identical partials till we reach partial 30, which is the sixth of the lower and fifth of the upper note, and is frequently so little developed and indistinct, that it makes no impression. Also, observe the beating partials 24 and 25, both lower than the common partial. One well qualified to make the assertion told me lately that no one could tune a minor Third. And certainly all manner of different ways of taking it exist, where there are not fixed tones to guide. It would scarcely have come into use had not the major Third and Fifth already existed and rendered it inevitable.

21. *Considence and Dissidence.*—Now observe that when the relative pitches here assumed, are tuned with perfect accuracy, if all these six notes were played together, we should hear only the fundamental with a different disposition in the loudness of the partials, and with many higher partials than would ordinarily be heard, and hence with a changed quality of tone. My instrument has just been tuned by an excellent tuner, but he had only his ear to guide him, and hence it is not quite accurate, though it will serve quite well enough to give you a notion of the result. I will play in succession note 1, and then the groups 1 2, 1 2 3, 1 2 3 4, 1 2 3 4 5, 1 2 3 4 5 6. Every partial you see fits properly into its place, and the partials which ought to "sit together," on the same rank in the fundamental, and which I hence propose to call *consident* partials, are really consident and hence do not beat. But if I slightly alter any of the relations, then the consident partials will be torn from their seats and become *dissident* and give rise to beats. Thus, change 5 into 5^{1}, which is what is done by the Greek change, and the column of note 5, now consisting of partials 5, 10, 15, 20, 25, 30, will have the partials $5\frac{1}{16}$ (beating seriously with partial 5 in note 1), $10\frac{1}{8}$ (beating with partial 10 in note 2), $15\frac{3}{16}$ (beating with partial 15 in note 3), $20\frac{1}{4}$ (beating with partial 20 in note 4), $25\frac{5}{16}$ (which has no audible partial to beat with, but which does not materially improve the beats of partials 25 and 24), and $30\frac{3}{8}$ (beating with partial 30 in note 6). Hence directly I use the Greek major Third notes 4 and $5\frac{1}{16}$, or, in whole numbers, 64 and 81, a series of harsh beats arise which are just sufficiently fast not to be readily counted but to be quite insupportable. Listen while I play the groups 1 2 3 4, 1 2 3 4 $5\frac{1}{16}$, 1 2 3 4 $5\frac{1}{16}$ 6. You hear the beats more strongly if I take

only the notes 4 5 6 at a lower pitch and then 4 $5\frac{1}{16}$ 6 at the same pitch. This is then the reason why the Greeks had no harmony. They had not fulfilled the conditions of harmony, which requires consident partials, and can only be formed from the numbers 2, 3, 5* and their products, taken as the relative pitch of the compound notes used.

22. *Just Intonation for Harmonious Scales.*—Let us apply this principle to the Greek scale. Write it as a series of exactly perfect Fifths taken upwards, or Fourths taken downwards, taking care to keep all the notes within the Octave c' to c'', although for convenience, as the precise Octave is of no consequence, Capitals only are used in writing, thus,

F C G D | A E B

Now here we find that the A as a major Third to F is too sharp by exactly $\frac{81}{80}$ of its pitch, and similarly E too sharp as a major Third to C, and B as a major Third to G, by the same fraction of the lower pitch.† Listen while I play F A, C E, G B in the Greek scale. Now let me take notes just flatter than these by one "comma," as the interval $\frac{81}{80}$ is called, and write these by putting an inferior 1 after the letters, thus A_1, E_1, B_1. Observe the difference of effect, while I pass from FA to FA_1. That great unbearable roughness has entirely vanished, because the dissident partials have been rendered consident. We may write this thus,

F C G D | A E B
A_1 E_1 B_1 |

Now this seems a very little change in the scale, but it is really a very great one. The scale with A_1 E_1 B_1 is called "just" because the relative pitches are *exactly* those required for avoiding dissident beats in the major Thirds. But an inconvenience at once arises. DA_1 is not a Fifth. Listen while I pass from DA to DA_1. The harshness of DA_1 or notes 2 $3\frac{3}{80}$, beating with 6 the third partial of the first and $6\frac{3}{40}$ the second partial of the second, is even worse than that of the bad major Thirds. To avoid this we are obliged to have a second form of D, as D_1. Observe while

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* Some persons admit 2, 3, 5, 7. But as partials 7 and 8 are within beating distance, the chord with notes 4 5 6 7, known as the chord of the harmonic Seventh, is not quite free from roughness even at the highest pitches. It is, however, greatly better than the usual substitute with the notes 4 5 6 $7\frac{1}{9}$, known as the chord of the dominant Seventh, and always reckoned as a discord. The harmony which stops at 2, 3, 5, or is founded on the Octave, Fifth, and Major Third, has been named Tertian Harmony; that which involves the Harmonic Seventh, Septimal Harmony. See my Appendix to Helmholtz, p. 660. The Tertian harmony alone is here considered, and is explained in the following articles.

† Let the names of the notes be used for their pitches. Then G is $\frac{3}{2}$C, D a Fourth down is $\frac{3}{4}$G or $\frac{9}{8}$C, A is $\frac{3}{2}$D or $\frac{27}{16}$C, E a Fourth down is $\frac{3}{4}$A or $\frac{81}{64}$C, that is, $\frac{81}{80} \times \frac{5}{4}$C. Hence, as the proper major Third would be $\frac{5}{4}$C, this is too sharp by exactly $\frac{81}{80}$ of the proper pitch.

I play D A, D A_1, $D_1 A_1$. The first and last are good, the second very bad. Our scale now becomes

$$\begin{array}{c|cccc|ccc} & F & C & G & D & A & E & B \\ D_1 & A_1 & E_1 & B_1 & & & & \end{array}$$

but what are we to do with the two forms D, D_1, which are far too nearly alike to be used in succession? It will soon be evident that to produce agreeable effects we must use D_1 with F and A_1, and D otherwise, and in mere melody we may use D in ascending and D_1 in descending. Listen while I play a scale with this difference, namely C D E_1 F G A_1 B_1 c, c B_1 A_1 G F E_1 D_1 C. And now I will play the first bars of *God Save the Queen* harmonised with this scale, as a contrast with the Greek.

23. *The Conditions of Harmony.*—We have seen roughly, therefore, that harmoniousness, or absence of dissident beats, when several notes or compound tones are played together, can be produced by selecting some of the notes of a scale like that just found, where we first take 4 notes forming Fifths (or Fourths) as F, C, G, D, then depress the last by a comma, forming D_1 and taking three additional notes also forming Fifths, thus D_1 A_1 E_1 B_1. Any set of notes played together forms a "chord." If no dissident beats are produced, we call the chord a "concord"; but if such beats arise, we call the chord a "discord." A concord differs in some respects from the results previously produced (art. 21) by playing together a fundamental and its harmonics (art. 13). In that case we heard the effect of a fundamental note of the same pitch with a new quality of tone. In chords the fundamental note is usually omitted, or played softly, while some of the harmonics are often omitted, and others greatly increased in power. The new quality of tone thus "artificially" produced, differs greatly from any "natural" quality of tone that can be obtained by playing any compound note by itself, and it is the succession of such artificial qualities of tone or chords which produces "harmony." When the different notes of a chord are taken by different voices they are called "parts," and the loudest or most prominent part heard in passing from chord to chord produces the "melody" or "subject" to which the other parts form the "harmonies." Each part, however, in skilfully arranged harmonies has a melody of its own, only less conspicuous because less loud. At present the highest part usually "leads" and gives "the air," and the lower parts form its harmonies, or are reduced to an instrumental "accompaniment."

Without going into the beautiful arrangements which have been worked out by the musicians of harmony during the last six or seven hundred years,* it is worth while considering the conditions on which such arrangements can be most harmoniously carried out, that is, what notes must be selected to form scales. This may be better understood by the following :—

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* The Cuckoo Song, or "Sumer is i-cumen in," in the Harleian Manuscript, No. 978, folio 10*b*, in the British Museum, printed in facsimile as a frontispiece to Mr. W. Chappell's "Popular Music in the Olden Time," and printed in an exact translation into modern musical notation on p. 426 of my "Early English Pronunciation," is, as Mr. Chappell says, "not only one of the first English songs with or without

DIAGRAM OF HARMONICS OF B FLAT.

times	G	G	D	G	B_1	D	G	A	B_1	D	F_1*sh*	G	A	B_1	D	D_2*sh*	E	F_1*sh*
times	C	C	G	C	E_1	G	C	D	E_1	G	B_1	C	D	E_1	G	G *sh*	A	B_1
times	F	F	C_1	F	A_1	C	F	G	A_1	C	E_1	F	G	A_1	C	C_2*sh*	D	E_1
nce	B*fl*	B*fl*	F	B*fl*	D_1	F	B*fl*	C	D_1	F	A_1	B*fl*	C	D_1	F	F_2*sh*	G	A_1
tch	1	2	3	4	5	6	8	9	10	12	15	16	18	20	24	25	27	30

Here the bottom line of numbers gives the relative pitch of the notes in each of the lines above it. It must be remembered that these are all really musical notes, or compound tones, and hence each is made up of a series of partials as in the former diagram (p. 19). Taking the beginning of the bottom line of notes, B *flat*, as the first fundamental, with which all the other pitches are compared, the line of figures give the pitches of the notes in this bottom line of notes. Then 3 times those pitches give the pitches of the notes in the second line beginning with F; 9 times the first pitches, or 3 times the second, give those in the third line of notes beginning with C; and 27 times the first pitches, or 9 times the second, and 3 times the third, give the pitches of the notes in the top line beginning with G. The pitch of the last F_1 *sharp* in the top line, then, is 27 times 30 times, or 810 times the pitch of the first B *fl* in the bottom line of notes, so that all the notes may be considered as "harmonics" of B *fl* in the sense explained on p. 18. But with that fact our ears have little concern. Our sense of hearing deals only with the actual relative pitch, the quality of tone produced, and the beats and differential tones called into existence. In fact the harmonics are seldom taken at the high pitches which are here marked. But from them notes are produced several octaves lower, of which the pitch therefore is not a multiple of the pitch of B *fl*, but a fraction of that pitch. Such notes are not its harmonics at all, but only tones of which B *fl* or some of its harmonics are also harmonics. The real musical value of harmonics, over which some theorists run wild, arises from the fact that they furnish a set of artificial partials which produce the smallest amount of beats. For this

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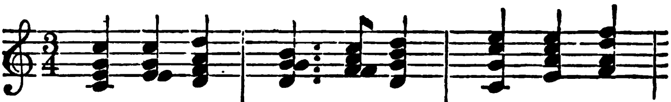
music, but the first example of counterpart in six parts, as well as of fugue, catch, and canon; and at least a century, if not two hundred years, earlier than any composition of the kind produced out of England." The date of the MS is fixed by Sir F. Madden as about A.D. 1250. It was written at Reading, in the monastery founded by Henry I., in 1125. The words are a simple pastoral in English, with a Latin hymn below to the same notes. It is a four-part harmony, with a low part for two extra bass voices, which produce harmonies of a nature not now allowed, as a kind of burden to the rest. Harmony exists only in Western Europe. The scales of the Greeks, Arabs, and Indians were, and are, as we have seen, unsuited for harmony.

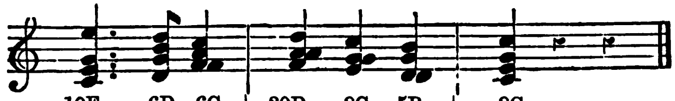
purpose the harmonics 7, 11, 13, 14, 17, 19, 21, 22, 23 have been omitted from the list. None of them form part of ordinary music, and there is no occasion here to enter upon the long discussions respecting 7, 14, 21 in especial. (See note * to p. 23.)

To shew more distinctly the meaning of this method of forming chords, I will take the arrangement of *God Save the Queen*, which I have just played, and underneath each chord I will write the names and relative pitches of its notes in the same order, so that by referring to the preceding diagram it will be seen what selection of the harmonics of B *fl*, F, C, G has been made in each case.

RELATIVE PITCH OF CHORDS IN "GOD SAVE THE QUEEN."



Chords	1	2	3	4	5	6	7	8	9
	8C	8C	20D_1	5B_1	6C	6D	5E_1	30E_1	24F
	6G	6G	15A_1	4G	5A_1	5B_1	4C	24C	20D_1
	5E_1	5E_1	12F	4G	4F	4G	3G	20A_1	15A_1
	4C	5E_1	10D_1	3D	4F	3D	2C	15E_1	12F
harmonics of	C	C	B*fl*	G	F	G	C	F	B*fl*



Chords	10	11	12	13	14	15	16
	10E_1	6D	6C	20D_1	8C	5B_1	8C
	6G	5B_1	5A_1	15A_1	6G	4G	6G
	5E_1	4G	4F	15A_1	6G	3D	5E
	4C	3D	4F	12F	5E_1	3D	4C
harmonics of	C	G	F	B*fl*	C	G	C

It will be observed that as long as the notes of the first harmonious scale derived from the Greek (on p. 23) are retained, namely, C D E_1 F G A_1 B_1 C, the chords are all made from the harmonics of three notes F, C, G; but that when D_1, the second form of D (on p. 24), is introduced, the harmonics are those of B *fl*.

24. *Major and Minor Chords.*—By examining the diagram of harmonics on p. 25, it will be seen that all the numbers are made up of 1, 3, and 5, separately, or multiplied together, as 3 times 3 or 9, 3 times 5 or 15, 5 times 5 or 25, and these results multiplied any number of times by 2. Also we see that the name of the note is preserved, however many times its number may be multiplied by 2. Thus in the C line, 1, 2, 4, 8, 16 are all C, and 3, 6, 12, 24 are all G, and 5, 10, 20 are all E_1. Each multiplication by 2 is equivalent to taking the note an octave higher. The effect

to the ear of these higher octaves is very different from that of the lower notes, both in succession and in combination. This is shewn in ordinary musical notations by changing the position of the small black circle or oval which marks the note as the octave changes, but we give the same name to the note.

Let us agree to disregard this change of octave for the moment, so that the series of harmonic numbers becomes 1, 3, 5, 9, 15, 25. Then, on examining the chords on p. 26, it will be seen that they fall into two classes. Chords 1, 2, 4, 5, 6, 7, 10, 11, 12, 14, 15, 16 are made up of the numbers 1, 3, 5, each more or less multiplied by 2; and chords 3, 8, 9, 13 are made of the numbers 3, 5, 15, each more or less multiplied by 2. The effect of these two kinds of chords is very different. When the notes are taken "close," that is, very near to each other in the scale, the first chords have comparatively small numbers to mark the relative pitch of their component notes, and hence we feel more distinctly the relation of these chords to the "fundamental" (p. 18), a circumstance which gave rise to the conception of a "fundamental bass" for every chord. These chords, depending on 1, 3, 5, in which the relative pitch can be expressed in small numbers, have also a finer and more musical effect, and are known as "Major Chords," because, when taken as close as possible, as in $4C\ 5E_1\ 6G$, the lowest interval $4C\ 5E_1$ is a major Third. But the chords depending on 3, 5, 15, when taken close, as they always are in the example here chosen, depend upon higher numbers, so that their relation to the fundamental is not so easily felt. Indeed great disputes have arisen respecting what should be taken as the fundamental bass of chords of this sort, which have generally a melancholy, weird effect, and in certain positions of the notes (that is, for certain multiplications by 2) are apt to become inharmonious. These are called "Minor Chords," because when taken close, in the form $10E\ 12G\ 15B_1$, the lowest interval $10E_1\ 12G$, which is the same as $5E_1\ 6G$ is a minor Third.

Now, as the numbers when unmultiplied by 2, are on the whole 1, 3, 5, 9, 15, 25, 27, so far as the diagram on p. 25 extends, it may seem strange that we selected first three 1, 3, 5 for the major chord, and 3, 5, 15 for the minor chord, neglecting 9, 25, and 27. But if we take 1, 3, 5, 9, or 3, 5, 9, which in their closest forms are $5E_1\ 6G\ 8C\ 9D$, or $9D\ 10E_1\ 12G$, we come upon the beating combinations 8 and 9, or 9 and 10, on which the ear cannot rest with pleasure. If we take 1, 3, 5, 15, which in the closest position become $8C\ 10E_1\ 12G\ 15B_1$, the second partial of 8C, or 16C, will beat violently with $15B_1$. If we take 9, 15, 25, or in the closest form 18D, $25G_2$*sh*, $30B_1$, we find that the third partial of 18D (that is 54A) beats strongly with the second partial of $25G_2$*sh* (that is, $50G_2$*sh*). All such combinations consequently give rise to "discords," whereas the major and minor chords are evidently "concords," and are the only possible concordant combinations of three notes with different names. Brought into their closest positions, after rejecting octaves, these chords are characterized thus:—

Major chords consist of any note as C, its major Third as E_1, and its Fifth as G.

Minor chords consist of any note as E_1, its *minor* Third as G, and its Fifth as B_1.

All other chords are discords.

25. *Derivation of Major and Minor Scales.*—The reduction of the harmonics to be considered to 1, 3, 5, 9, 15, 24, as just indicated, points out a method of contracting the first diagram of harmonics on p. 25 which will be found of great service. Let 3^2 mean 3 times 3, or two multiplications by 3; and 3^3 mean 3 times 3^2, or three multiplications by 3; and so on. Neglect the multiplications by 2, so that any number of them can be made. Then the whole of this diagram of harmonics may be presented as follows, giving the lowest pitch of each note with a new name.

CONTRACTED TABLE OF HARMONICS OF B FLAT.

3^6	E			729
3^5	A			243
3^4	D	F_1*sh*		81
3^3	G	B_1	D_2*sh*	27
3^2	C	E_1	G_2*sh*	9
3	F	A_1	C_2*sh*	3
1	B*fl*	D_1	F_2*sh*	1
	1	5	$5^2=25$	

The note above any note, when taken as close as possible, is its Fifth, thus E_1 is the Fifth above A_1.

The note to the right of any note, when taken as close as possible, is the major Third above it, thus C_2*sh* is the major Third above A_1.

A note obliquely above any note to the left is a minor Third above it, thus C is a minor Third above A_1.

Hence, to find the names of the notes in a major chord, which in its closest position begins with A_1, we go one step to the right for the major Third C_2*sh*, and then one step obliquely to the left for the minor Third E_1 above that C_2*sh*, which will, as we see by the diagram, be the Fifth E_1 of the original note A_1. The major chord of A_1 is therefore A_1 C_2*sh* E_1, in which A_1 is called the First, C_2*sh* the Third, and E_1 the Fifth, from their relative position in the scale of A_1 major.

To form the minor chord beginning with A_1, we go obliquely to the left for C, the minor Third above A_1, and then horizontally to the right for the major Third E_1 above this C; which is also the Fifth E_1 above the original note A_1.

Hence, taking any Fifth as A_1 E_1, in which A_1 is again called the First, and E_1 the Fifth, from their relative position in the scale of A_1, we complete it into a major chord by going one step horizontally to the *right* of the *bottom* note A_1 for C_2*sh*, the major Third, or complete it into a minor chord by going one step horizontally to the *left* of the *top* note E_1 for C the minor Third.

C E_1
A_1 C_2sh

These notes, which I have extracted in the margin, form what I call the "harmonic cell" of A_1, because they are harmonics forming the major and minor chords of A_1, for the same Fifth A_1 E_1. Now, evidently by taking the Fifth B_1 above the upper note, we might form another cell, as in the margin. This would form the harmonic cell of E_1, containing the major and minor chord of E_1 for the same Fifth E_1 B_1.

G B_1
E_1 G_2sh

Now, piling this cell on the top of the former, as in the margin, we get a combination of seven notes, which I call the "heptad" of E_1, in which the six notes all grouped about E_1 are concordant with it, and which contains the only three major chords A_1 C_2sh E_1 and E_1 G_2sh B_1, with C E_1 G, of which E_1 can form one of the notes; and also the only three minor chords, A_1 C E_1 and E_1 G B_1, with C_2sh, E_1, G_2sh, of which E_1 can form one of the notes. It also contains all those discords of three notes of which two are concordant with E_1 but discordant with each other, as the major trine C E_1 G_2sh (composed of two major Thirds, C E_1, E_1 G_2sh) and the minor trine C_2sh E_1 G (composed of two minor Thirds, C_2sh E_1, E_1 G).

G B_1
C E_1 G_2sh
A_1 C_2sh

But it is quite evident that we could also form from the original cell A_1, the harmonic cell of D_1, of which A_1 is the Fifth, as in the margin on the left. If this cell were placed below the cell of A_1, as in the margin on the right, we should obtain the heptad of A_1, having properties related to A_1 similar to those related to E_1 in the heptad of E_1.

F A_1
D_1 F_2sh

C E_1
F A_1 C_2sh
D_1 F_2sh

Moreover, we see that these two heptads have the common cell of A_1, as in the margin. This cell closely connects the heptads. Suppose, then, we pile the three cells upon one another, we shall obtain the annexed collection of ten notes, which I call the "decad" of A_1, because it rises out of the cell of A_1. The decad of A_1 clearly consists of the connected heptads of A_1 and E_1 (its Fifth), and of their three constituent cells. Of these, the lowest cell, or that of D_1, the Fifth below A_1, having A_1 as its Fifth, is called the "sub-dominant" cell; the middle cell, or that of A_1, having A_1 as its First, is called the "tonic cell"; and the highest cell, or that of E_1, the Fifth above A_1, is called the "dominant cell." This collection of notes must evidently be the key to all harmony.

C E_1
A_1 C_2sh

G B_1
C E_1 G_2sh
F A_1 C_2sh
D_1 F_2sh

Keeping the four central notes D_1 A_1 E_1 B_1, the three on the right F_2sh C_2sh G_2sh give a set of 7 notes which contain three major chords D_1 F_2sh A_1, and A_1 C_2sh E_1, and E_1 G_2sh B_1, called from their cells the subdominant, tonic, and dominant major chords; with two connecting minor chords, F_2sh A_1 C_2sh and C_2sh E_1 G_2sh. These five chords furnish the natural harmonies of the harmonic scale of A_1 major, which, when all the notes are reduced to the same Octave, and sounded in succession, consists of the following notes :—

A_1 B_1 C_2sh D_1 E_1 F_2sh G_2sh A_1.

The generating principle of an harmonic scale is the existence of three major chords, the subdominant, tonic, and dominant, so related that the First and Fifth of the tonic form the Fifth of the subdominant and the First of the dominant chord respectively.

Similarly, if we kept the same central column D_1 A_1 E_1 B_1 forming the Fifths, and took the three notes to the left, we should get three minor chords D_1 F A_1, A_1 C E_1, E_1 G B_1, related to each other as subdominant, tonic, and dominant, and also two connecting major chords F A_1 C, C E_1 G. These five chords furnish the natural harmonies of the natural minor scale of A_1, which, with the exception of one note (D_1 in place of D), consists of the same notes as the major scale of C. But this one note constitutes a great difference in the character of the harmonies, for whereas the natural minor scale of A_1 has the three related minor chords and two connecting major chords just mentioned, the major scale of C has three related major chords FA_1C, CE_1G, GB_1D, and two connecting minor chords A_1CE_1, E_1GB_1. Hence the two chords GB_1D and D_1FA characterise and distinguish the major scale of C and the natural minor scale of A_1, or the natural relative minor scale of C, as it is called, and the other four chords are identical in each scale. On the piano, where D and D_1 are confounded, this character is lost, and the two scales seem to consist of the same notes, and only to be distinguished by the predominant use of major chords in the one and minor chords in the other, for which use no intelligible reason is furnished by the notes themselves.

But modern ears do not approve of this natural minor scale, except in descending, thus—

A_1 G F E_1 D_1 C B_1 A_1.

When ascending, musicians now prefer to make the scale end like the major scale of A_1, that is, either as—

A_1 B_1 C D_1 E_1 F G_2sh A_1,

where G_2sh, on the right of the contracted table of harmonics (p. 28), is taken in place of G on the left, and hence the three generating chords of the scale are the minor subdominant D F A_1, the minor tonic A_1 C E_1, and the major dominant E_1 G_2sh B_1.

Again, as the step F to G_2sh is found to be rather troublesome to the singer, the last half of the scale is often made exactly like the major scale, thus

A_1 B_1 C D_1 E_1 F_2sh G_2sh A_1.

Here the F_2sh is evidently derived from the right of the contracted diagram, in place of the F on the left, so that the three generating chords of the scale are now the major subdominant D_1 F_2sh A, and the minor tonic A_1 C E_1, and the major dominant E_1 G_2sh B_1. Hence there are now three usual forms of the minor scale—(1) descending, all three generating chords minor; (2) ascending, dominant chord major, the others minor; (3) ascending, tonic chord minor, the others major. Many other scales, formed by varying the three generating chords, are also known to musicians.

26. *Chords of the Scale.*—The following is a very convenient way of viewing the major and minor chords and principal discords in any scale. Let (+) between two notes indicate that they make a just major Third, (−) that they make a just minor Third, and (|) that they make a Greek minor Third, which is a comma less than a just minor Third, and is a discord, and call the sign (+) *major*, the sign (−) *minor*, and the sign (|) *Greek*. Then we may write the major and minor scales of A_1 thus, proceeding by Thirds. The scale lies between the | |, but is continued for two notes each way beyond.

MAJOR SCALE OF A_1.

$$G_2sh - B_1 \mid D_1 + F_2sh - A_1 + C_2sh - E_1 + G_2sh - B_1 \mid D_1 + F_2sh.$$

DESCENDING MINOR SCALE OF A

$$G + B_1 \mid D_1 - F + A_1 - C + E_1 - G + B_1 \mid D_1 - F.$$

FIRST ASCENDING MINOR SCALE OF A_1.

$$G_2sh - B_1 \mid D_1 - F + A_1 - C + E_1 + G_2sh - B_1 \mid D_1 - F.$$

SECOND ASCENDING MINOR SCALE OF A_1.

$$G_2sh - B_1 \mid D_1 + F_2sh - A_1 - C + E_1 + G_2sh - B_1 \mid D_1 + F_2sh.$$

Then, in the major and descending minor scale, any three notes between the | | will form a major or minor chord, but any three of which one or two extend beyond the | | form a discord, and any four or five together, whether within or without the | |, form discords. In the first ascending minor scale we get the discordant major trine $C + E_1 + G_2sh$, and in the second ascending minor scale both this and the discordant minor trine $F_2sh - A_1 - C$, within the | |.

Besides these, the following discords are most used in music.

In the MAJOR SCALE OF A_1—

$E_1 + G_2sh - B_1 \mid D_1$, called the chord of the dominant or minor Seventh; or, omitting the lowest note,

$G_2sh - B_1 \mid D$, called the chord of the diminished Fifth;

$E_1 + G_2sh - B_1 \mid D_1 + F_2sh$, called the chord of the major Ninth; or, omitting the lowest note,

$G_2sh - B_1 \mid D_1 + F_2sh$, called the chord of the major Seventh;

$B_1 \mid D_1 + F_2sh$, called the chord of the added Sixth, for which is often substituted the minor chord $B_2 - D_1 + F_2sh$ which properly belongs to the subdominant scale of D_1 major.

In the FIRST ASCENDING MINOR SCALE OF A_1—

$E + G_2sh - B_1 \mid D_1 - F$, called the chord of the minor Ninth; or, omitting the lower note,

$G_2sh - B_1 \mid D_1 - F$, called the chord of the diminished Seventh.

Other more complex chords depending on modulations are purposely omitted. Each of these chords and discords must be carefully studied in their separate effects, taking the component notes in different Octaves. The variety of characteristic sound thus produced is quite wonderful.

27. *Modulation and Accidentals.*—As long as we keep either to a major or any of the forms of a minor scale, that is, as long as we

use only such notes as it contains, we are said to keep in one "key." But when we introduce notes, and hence chords, of another scale, we are said to "modulate." Now if we look at the contracted table of harmonics given in Art. 25, p. 28, we see that the last four lines, separated off by a cross line, contain the 10 notes of the decad of A_1, and two extra notes, D_2*sh* at the top, and B*fl* at the bottom, which are added as a preparation for modulation.* These 12 notes I call the "duodene" of A_1, of which it contains the decad. They correspond to the 12 notes on a piano (which, however, as we shall soon see, are designedly tuned imperfectly), and consist of three columns of notes forming Fifths, and four lines of notes forming major Thirds. The duodene of A_1 contains complete the major and three minor scales of A_1, the major scale of F, and the descending minor scale of C_2*sh*.

We see how new duodenes can readily be formed by erasing the bottom line, and adding a new line above; or erasing the top line and adding a new line below; or erasing a right (or left) hand column, and adding a new left (or right) hand one. The figures 1 and 2 below the notes in the second and third columns shew that the notes are 1 and 2 commas flatter than notes of the same name in the first column; thus D_1, A_1, E_1, are each a comma flatter than D, A, E, and F_2*sh* is a comma flatter than F_1*sh*, and so on. But within the duodene itself there are no two notes which differ by a comma. In this way the annexed diagram (in which *ssh* means "double sharp") will be found to contain the nine related duodenes of C, F, *Bfl*; E_1, A_1, and D_1, and G_2*sh*, C_2*sh*, F_2*sh*. A study of this diagram will shew the student with the greatest readiness the real relations of the keys which render modulation or change of key possible. This is a subject which lies beyond the scope of this lecture, but the principles on which modulation depends are easily stated.

DIAGRAM OF ELEMENTARY MODULATIONS.

B^1*fl*	D	F_1*sh*	A_2*sh*	C_2*ssh*
E^1*fl*	G	B_1	D_2*sh*	F_3*ssh*
A^1*fl*	C	E_1	G_2*sh*	B_3*sh*
D^1*fl*	F	A_1	C_2*sh*	E_3*sh*
G^1*fl*	B*fl*	D_1	F_2*sh*	A_3*sh*
C^1*fl*	E*fl*	G_1	B_2	D_3*sh*

Each Scale consists of three Generating Chords. Any one of these may be the generating chord of some other scale. The composer then can at will think himself into another scale of which this particular chord is one of the generating chords.

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* These two notes also allow of the formation of some of the pleasantest discords we have. They really approach very nearly to the harmonic chord 4, 5, 6, 7, but on the piano they do not differ from the chords of the minor Seventh. They are called the chords of the German Sixth, namely, B*fl* + D_1 − F with G_2*sh* (in place of *Afl*) and F + A_1 − C with D_2*sh* (in place of E*fl*). Omitting the Fifth in each chord, they become the chords of the Italian Sixth, B*fl* + D_1 with G_2*sh*, and F + A_1 with D_2*sh*. When these are introduced, there is an immediate modulation.

Each Note is part of six Chords (as shewn by the heptad on p. 29), and each of these chords may be considered as the generating chord of a new scale. Hence the composer, by retaining the one note and altering the two which form a concord in it, may pass into a scale of which this new chord is a generating chord, without occasioning any strange effect.

The four following modulations, which are the most common, will shew the action of this principle. They are all referred to the Notes and Generating Chords of the Original Scale of C major, F A_1 C, C E_1 G, G B_1 D.

1). Generating Chords of G, the Dominant of C, C E_1 G, G B_1 D, D F_1*sh* A. Two notes of the original scale are changed, the Fourth and Sixth F and A_1, which become F_1*sh* and A, the Seventh and Second of the new scale.

2). Generating Chords of F, the Sub-Dominant of C, B*fl* D_1 F, F A_1 C, C E_1 G. Two notes of the original scale are changed, the Seventh and Second B_1 and D, which become the B*fl* and D_1, the Fourth and Sixth of the new scale.

3). Generating Chords of A_1 minor, the Relative Minor of C.

Descending form D_1FA_1, A_1C E_1, E_1G B_1. Only one note of the original scale is changed, the Second of the old scale D which becomes D_1 the Fourth of the new scale.

First ascending form D_1FA_1, A_1C E_1, E_1 G_2*sh* B_1. Two notes of the original scale are changed, the Second and Fifth D and G, which become D_1 and G_2*sh*, the Fourth and Seventh of the new scale.

Second ascending form, $D_1$$F_2$*sh*$A_1$, A_1CE_1, E_1 G_2*sh*B_1. Three notes of the original scale are changed, the Second, Fourth, and Fifth, D, F, G, which become D_1, F_2*sh*, and G_2*sh*, the Fourth, Sixth, and Seventh of the new scale.

4). Generating Chords of C minor, the Tonic Minor of C.

Descending Form, FA^1*fl* C, CE^1*fl* G, GB^1*fl* D. Three notes of the original scale are changed, the Third, Sixth, and Seventh, E_1, A_1, B_1, which become E^1*fl*, A^1*fl*, and B^1*fl*, the Third, Sixth, and Seventh of the new scale.

First ascending Form, FA^1*fl* C, CE^1*fl* G, G B_1 D. Two notes of the original scale are changed, the Third and Sixth, E_1 and A_1, which become E^1*fl* and A^1*fl* the Third and Sixth of the new scale.

Second ascending Form, FA_1C, CE^1*fl* G, G B_1 D. One note of the original scale is changed, the Third E_1, which becomes E^1*fl* the Third of the new scale.

Hence the simplest modulations from any major scale are into the descending form of its relative minor or the second ascending form of its tonic minor. Other forms of modulation are omitted.

It is this longing for modulation, which has taken such a strong hold on musicians, that has occasioned the introduction of the "accidentals." Neglecting the alteration by a comma, which, as